THE 15TH RUSSIAN-CHINESE WORKSHOP ON SPACE WEATHER

September 9–13, 2024, Irkutsk, Russia

DIAGNOSTICS OF CORONAL PLASMA USING THE EXACT SOLUTION OF THE EVOLUTION EQUATION FOR SLOW MAGNETOACOUSTIC AND ENTROPY WAVES

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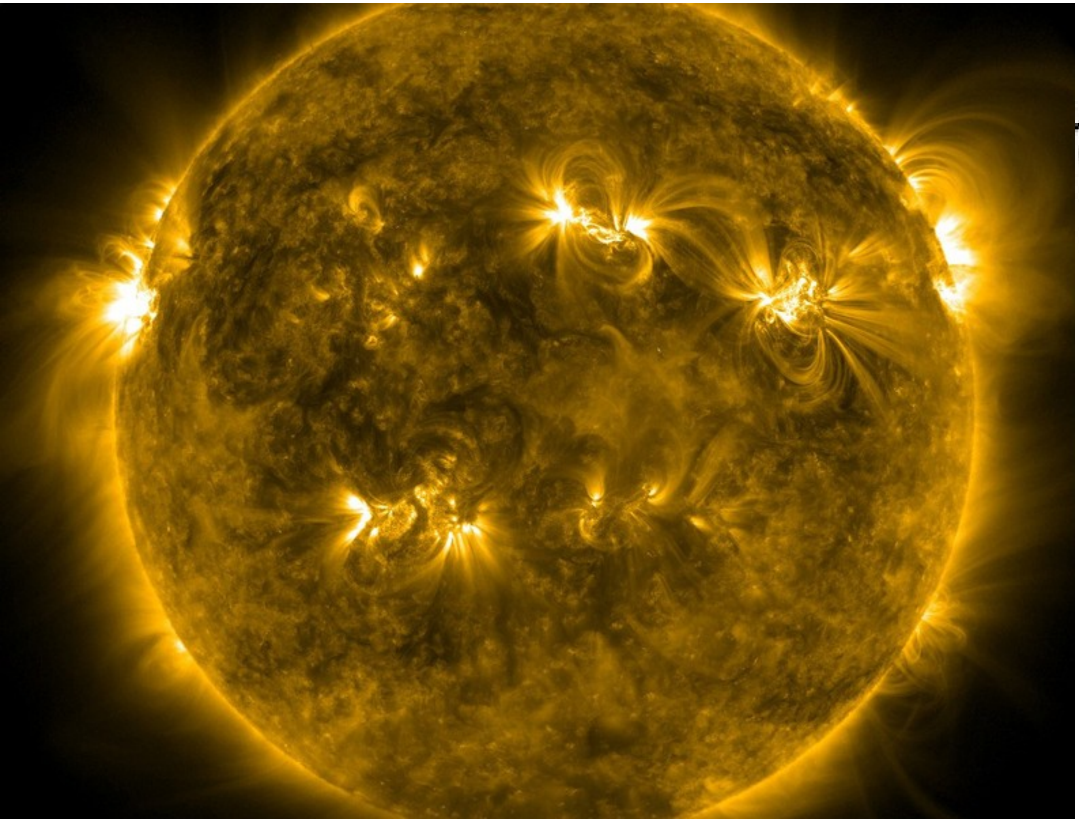
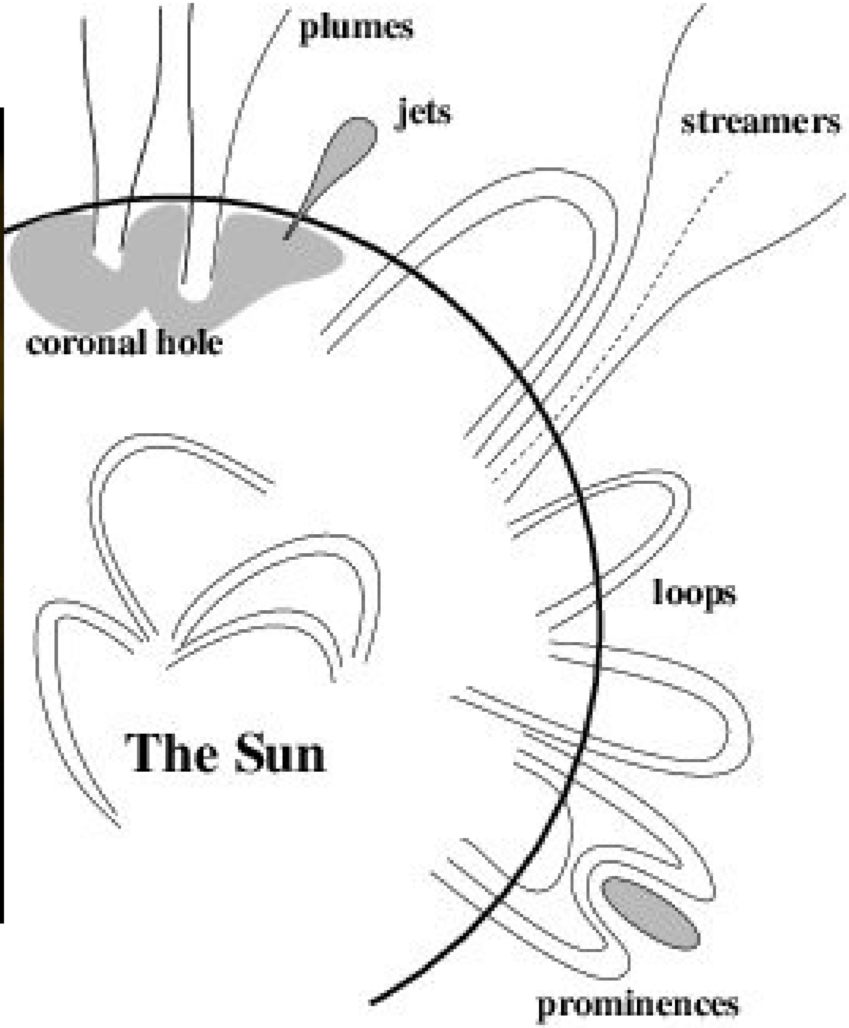
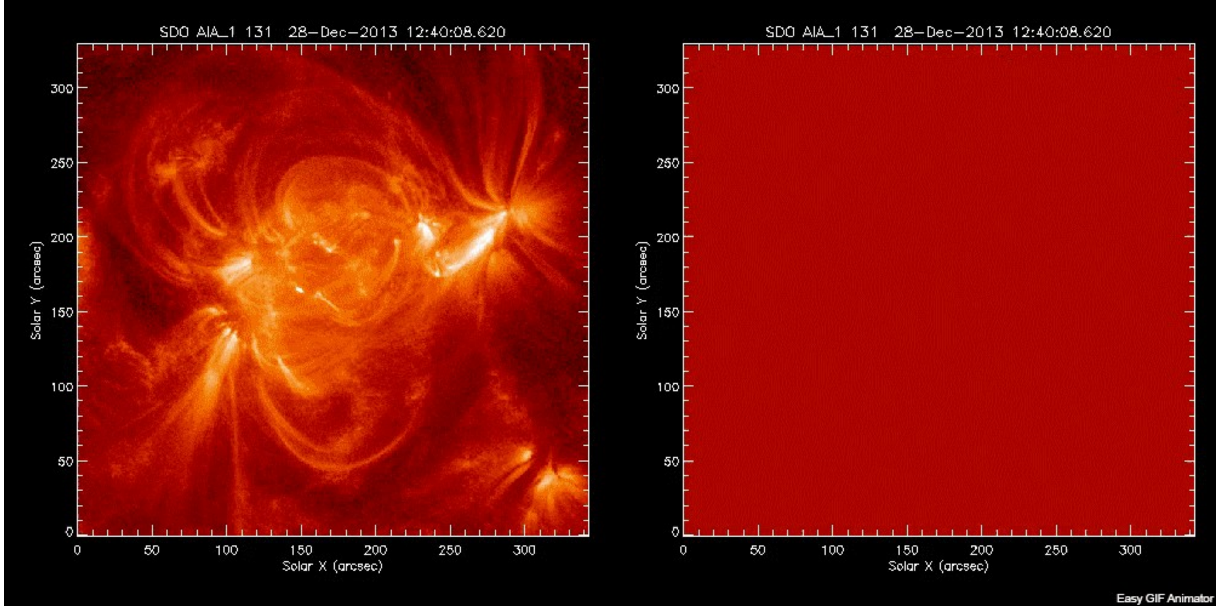
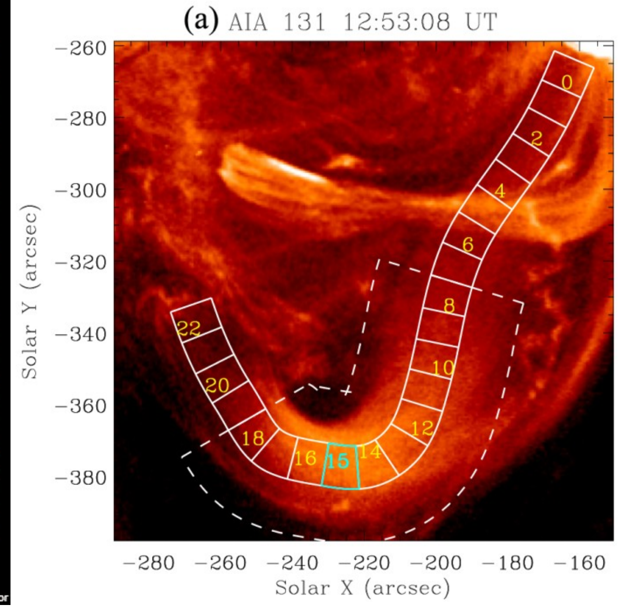
What's being observed?

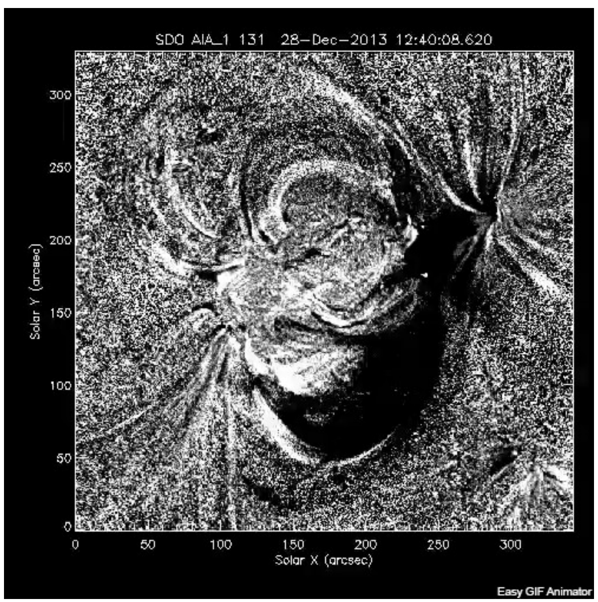
MHD-structures in solar atmosphere






SDO AIA_1 131 28-Dec-2013 12:40:08.620
SDO AIA_1 131 28-Dec-2013 12:40:08.620
Solar Y (arcsec)
Solar X (arcsec)
Easy GIF Animator


(a) AIA 131 12:53:08 UT
Solar Y (arcsec)
Solar X (arcsec)


SDO AIA_1 131 28-Dec-2013 12:40:08.620
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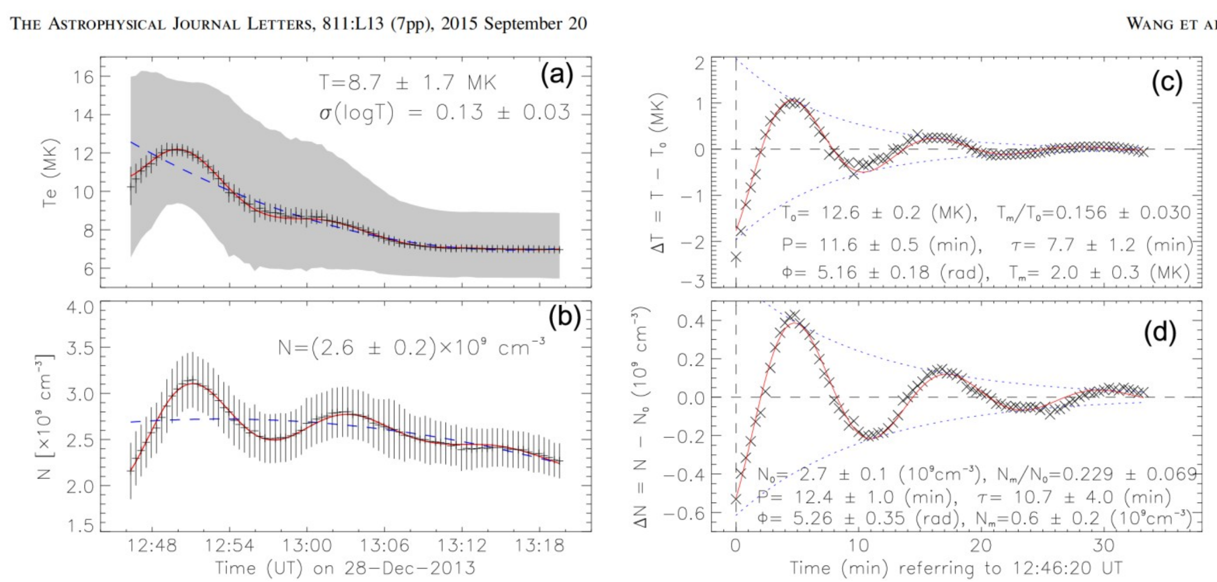

THE ASTROPHYSICAL JOURNAL LETTERS, 811:L13 (7pp), 2015 September 20
WANG ET AL.
(a)
T=8.7 ± 1.7 MK
σ(logT) = 0.13 ± 0.03
Te (MK)
(b)
N=(2.6 ± 0.2)×10⁹ cm⁻³
N [×10⁹ cm⁻³]
Time (UT) on 28-Dec-2013
(c)
ΔT = T − T₀ (MK)
T₀= 12.6 ± 0.2 (MK), Tₘ/T₀=0.156 ± 0.030
P= 11.6 ± 0.5 (min), τ= 7.7 ± 1.2 (min)
Φ= 5.16 ± 0.18 (rad), Tₘ= 2.0 ± 0.3 (MK)
(d)
ΔN = N − N₀ (10⁹ cm⁻³)
N₀= 2.7 ± 0.1 (10⁹cm⁻³), Nₘ/N₀=0.229 ± 0.069
P= 12.4 ± 1.0 (min), τ= 10.7 ± 4.0 (min)
Φ= 5.26 ± 0.35 (rad), Nₘ=0.6 ± 0.2 (10⁹cm⁻³)
Time (min) referring to 12:46:20 UT

How is it modeled?

Basic equations and assumptions

$$\rho \frac{dV_z}{dt} = -\frac{\partial P}{\partial z}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}(\rho V_z) = 0$$

$$C_{V\infty} \frac{dT}{dt} - \frac{k_Б \cdot T}{m\rho} \cdot \frac{d\rho}{dt} = -Q(\rho,T) + \frac{1}{\rho}\kappa_z \frac{\partial^2 T}{\partial z^2}$$

$$P = \frac{k_Б \cdot T \cdot \rho}{m}$$

$$Q(\rho,T) = L(\rho,T) - H(\rho,T)$$

- The influence of gravitational stratification is weak ()
- The effect of waveguide dispersion is weak ()
- The plasma is highly magnetized ()
- The effect of viscosity is negligible ()
- Plasma homogeneous along the waveguide

Evolutionary equation and dispersion properties

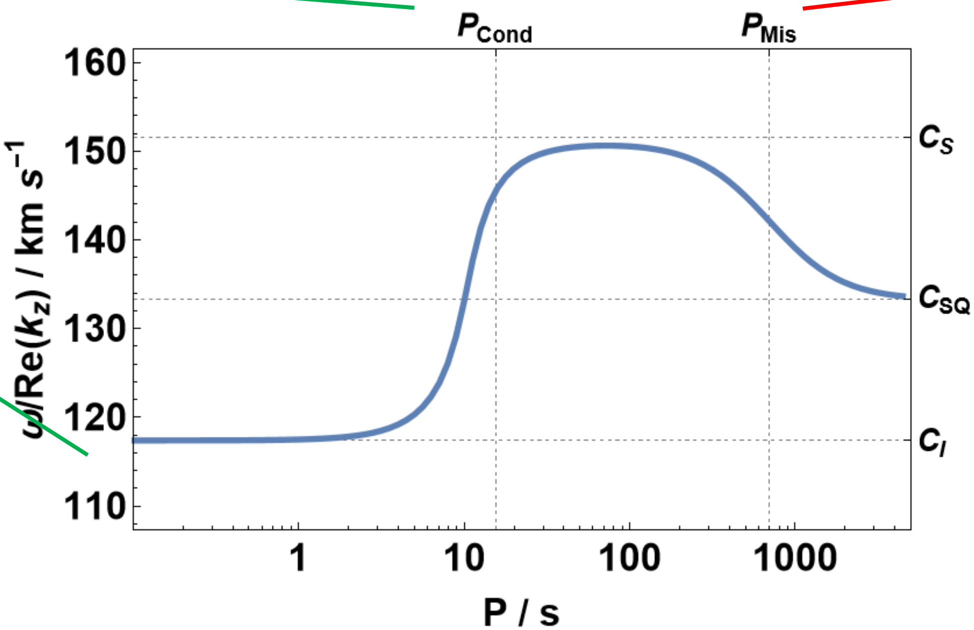
$$\frac{\partial^3 a_1}{\partial t^3} - c_S^2 \frac{\partial^3 a_1}{\partial t \partial z^2} = \frac{\kappa}{\rho_0 C_V}\left(\frac{\partial^4 a_1}{\partial z^2 \partial t^2} - c_{Si}^2 \frac{\partial^4 a_1}{\partial z^4}\right) - \frac{1}{\tau_V}\left(\frac{\partial^2 a_1}{\partial t^2} - c_{SQ}^2 \frac{\partial^2 a_1}{\partial z^2}\right)$$

- Damping
- Phase speed dispersion

- Damping or **Amplification!!!!!!!**
- Phase speed dispersion

$= \frac{2}{(\;)}$

$= \frac{2}{\sqrt{\;}}$



$= \sqrt{\frac{\;}{\;}}$ 0

$$c_{SQ} = \sqrt{\frac{\tau_V C_P}{\tau_P C_V}\frac{k_B T_0}{m}},$$

-

$$\tau_P = C_P T_0 / \left(Q_{0T} T_0 - Q_{0\rho}\rho_0\right),$$
$$\tau_V = C_V / Q_{0T},$$

Characteristic thermal conduction spatial scale

Thermal misbalance timescales (Defined by plasma heating and cooling rates)

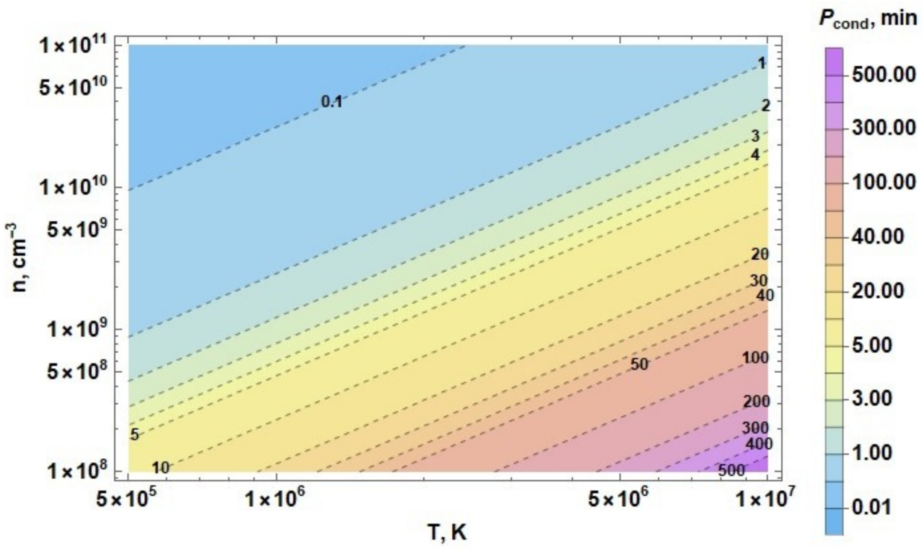
Zavershinskii et al 2023

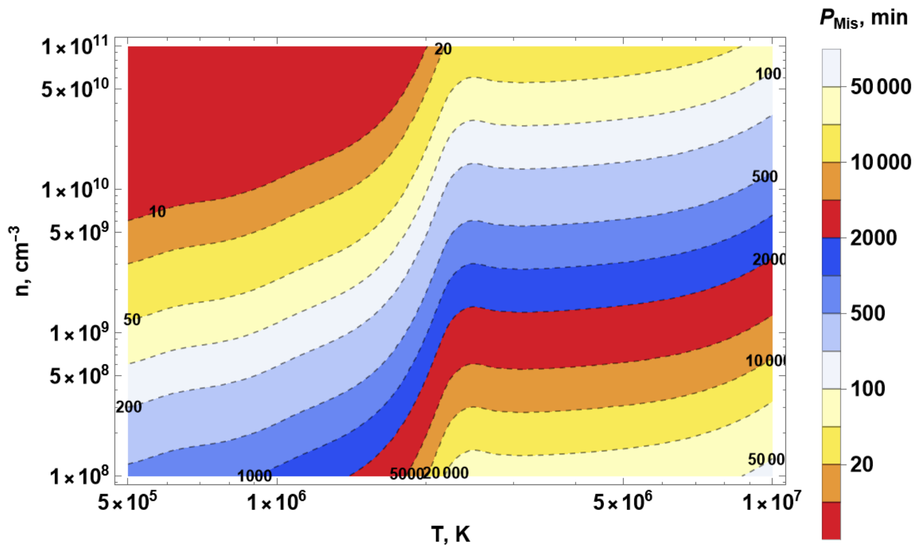
Zavershinskii et al 2021

Characteristic temporal scales

$$= \frac{2}{(\quad)}$$

$$= \frac{2}{\sqrt{\quad}}$$





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Reduced evolutionary equation

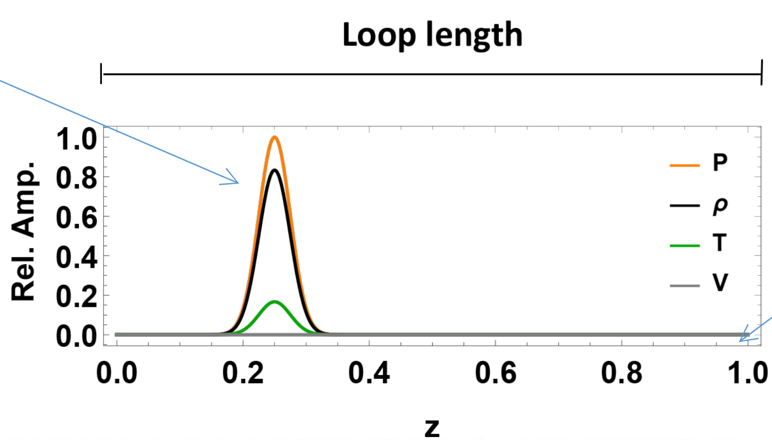
$$\frac{\partial^3 \widetilde{a}_j}{\partial \tilde{t}^3} - \gamma \frac{\partial^3 \widetilde{a}_j}{\partial \tilde{t} \partial \tilde{z}^2} = -\widetilde{d} \left(\frac{\partial^4 \widetilde{a}_j}{\partial \tilde{t}^2 \partial \tilde{z}^2} - \frac{\partial^4 \widetilde{a}_j}{\partial \tilde{z}^4} \right)$$

Here, we have introduced the dimensionless perturbation of plasma parameter $\widetilde{a}_j$. The index j defines the parameter under study. In other words, we use the following values $[\widetilde{a}_\rho = \rho_1/\rho_0]$ for density perturbation, $[\widetilde{a}_P = P_1/P_0]$ for pressure perturbation, $[\widetilde{a}_T = T_1/T_0]$ for temperature perturbation, and $[\widetilde{a}_u = u_1/c_{\mathrm{S}i}]$ for velocity perturbation. We also use dimensionless coordinate $[\tilde{z} = z/L]$, and time $[\tilde{t} = t/t_L, t_L = c_{\mathrm{S}i}/L]$. Here, L is the characteristic spatial scale.

$$\widetilde{d} = \frac{1}{\widetilde{\tau}_{\mathrm{cond}}} = \frac{t_L}{\tau_{\mathrm{cond}}}, \quad \tau_{\mathrm{cond}} = \frac{L^2 C_{\mathrm{V}} \rho_0}{\kappa},$$

Some initial signal
Of optional type and form

$$\rho = f_1(z,t)$$
$$P = f_2(z,t)$$
$$T = f_3(z,t)$$
$$V = f_4(z,t)$$



Reflecting boundaries

$$\frac{\partial \rho(0,t)}{\partial z} = \frac{\partial \rho(l,t)}{\partial z} = 0$$
$$\frac{\partial P(0,t)}{\partial z} = \frac{\partial P(l,t)}{\partial z} = 0$$
$$\frac{\partial T(0,t)}{\partial z} = \frac{\partial T(l,t)}{\partial z} = 0$$
$$V(0,t) = V(l,t) = 0$$

Solution of reduced evolutionary equation

$$a_\rho(z,t) = a_{\rho 0}(z,t) + \sum_{n=1}^{\infty} a_{\rho n}(z,t).$$

$$a_{\rho n}(z,t) = C_{1\rho n} e^{\omega_{\mathrm{EI}} t} \cos(kz) +$$

Entropy mode

$$C_{0\rho n} e^{\omega_{\mathrm{AI}} t} \left[\cos(\omega_{\mathrm{AR}} t + kz - \phi_{\rho n}) + \cos(\omega_{\mathrm{AR}} t - kz - \phi_{\rho n})\right],$$

Two Magnetoacoustic waves

$$C_{0\rho n} = \frac{\sqrt{C_{2\rho n}^2 + C_{3\rho n}^2}}{2}, \quad \phi_{\rho n} = \arctan\left(\frac{C_{3\rho n}}{C_{2\rho n}}\right).$$

$$a_{\rho 0}(z,t) = I_{10} = \frac{1}{l}\int_0^l \rho_{\mathrm{in}}(z,0)\mathrm{d}z.$$

Non-oscillating background

$$\begin{pmatrix} 1 & 1 & 0 \\ \omega_{\mathrm{EI}} & -\omega_{\mathrm{AI}} & \omega_{\mathrm{AR}} \\ \omega_{\mathrm{EI}}^2 & (\omega_{\mathrm{AI}}^2 - \omega_{\mathrm{AR}}^2) & -2\omega_{\mathrm{AR}}\omega_{\mathrm{AI}} \end{pmatrix} \begin{pmatrix} C_{1n} \\ C_{2n} \\ C_{3n} \end{pmatrix} = \begin{pmatrix} I_{1n} \\ I_{2n} \\ I_{3n} \end{pmatrix}.$$

$$I_{1n} = \frac{2}{l}\int_0^l \rho_{\mathrm{in}}(z,0)\cos(kz)\,\mathrm{d}z,$$

$$I_{2n} = \frac{2}{l}\int_0^l \left.\frac{\partial \rho(z,t)}{\partial t}\right|_{t=0} \cos(kz)\,\mathrm{d}z,$$

$$I_{3n} = \frac{2}{l}\int_0^l \left.\frac{\partial^2 \rho(z,t)}{\partial t^2}\right|_{t=0} \cos(kz)\,\mathrm{d}z.$$

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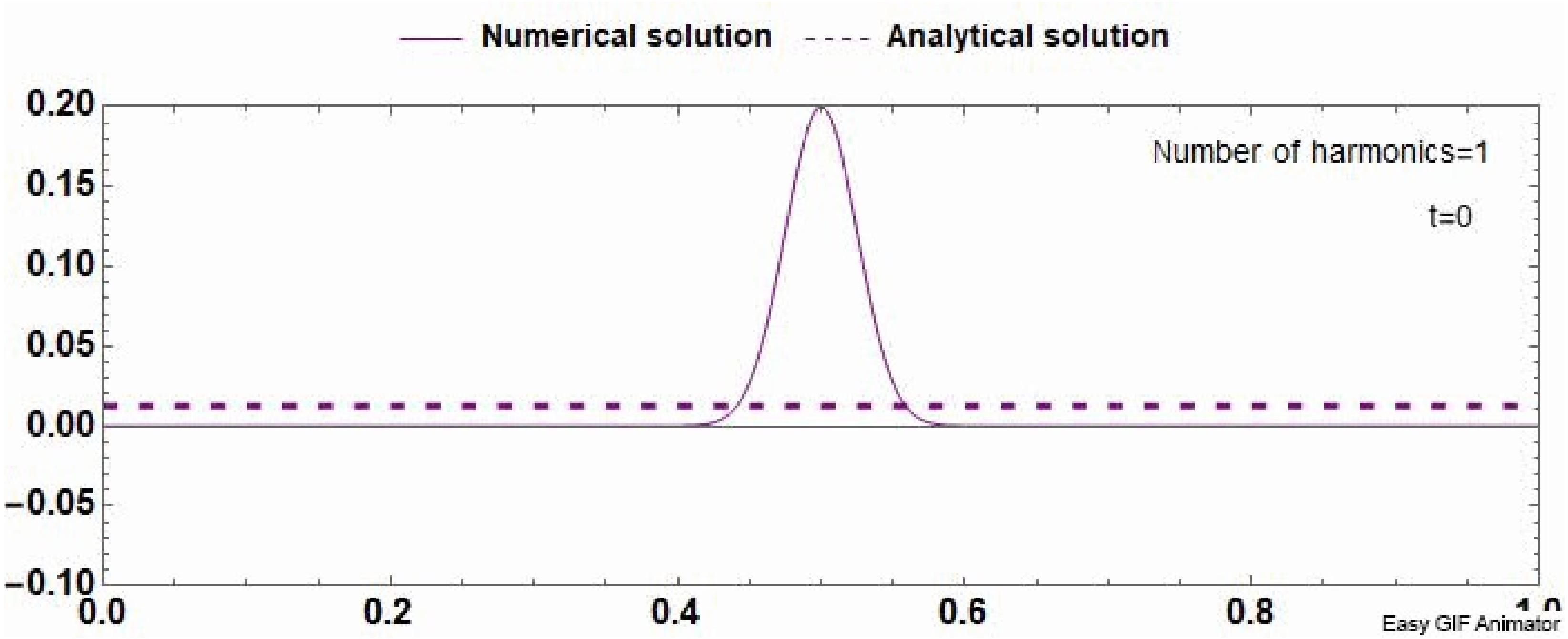
That's all nice, but..... Is it really working?

Comparison of analytical and numerical solutions

Relative density amplitude



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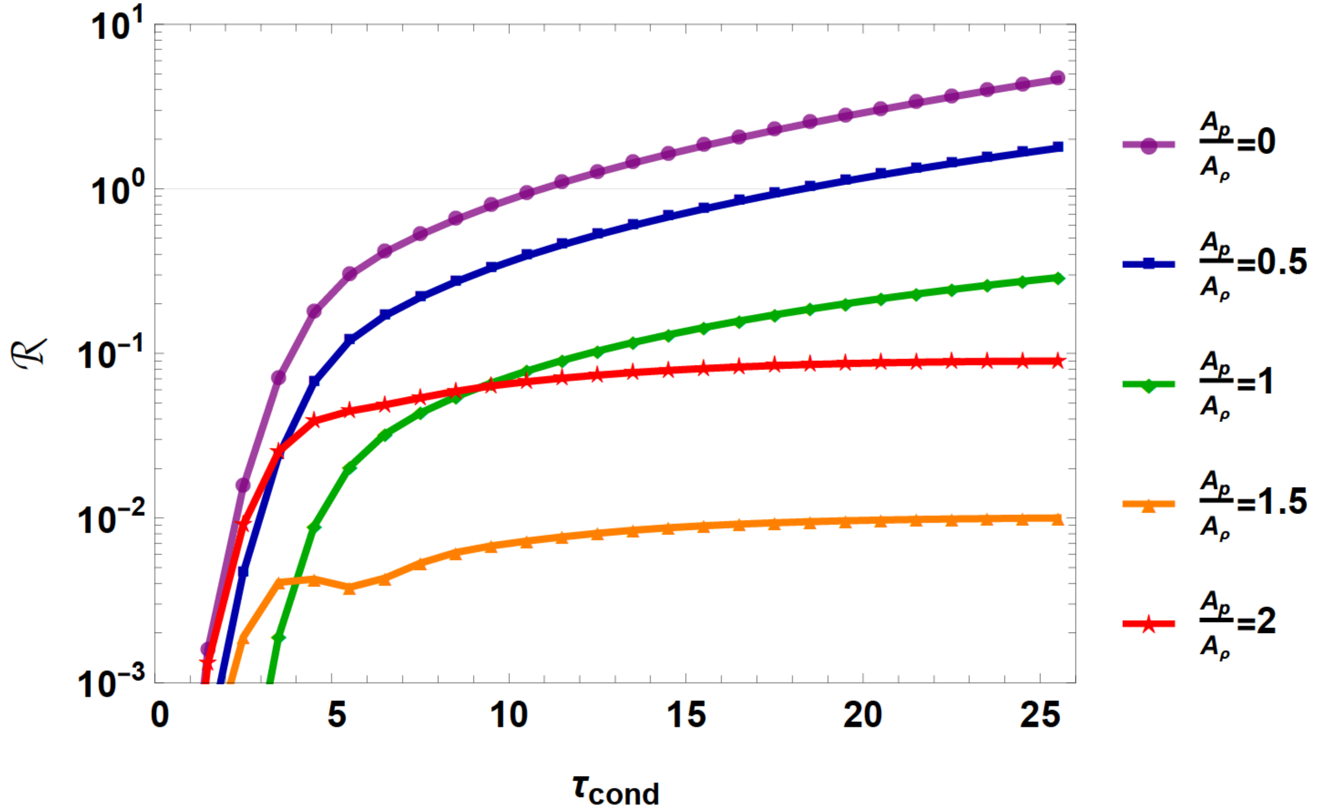
How to apply it?

Distribution of energy between modes

$$\mathcal{R} = \sum_{n=1}^{\infty} C_{1n}^2 / \sum_{n=1}^{\infty} 4C_{0n}^2 = \frac{Es}{As},$$

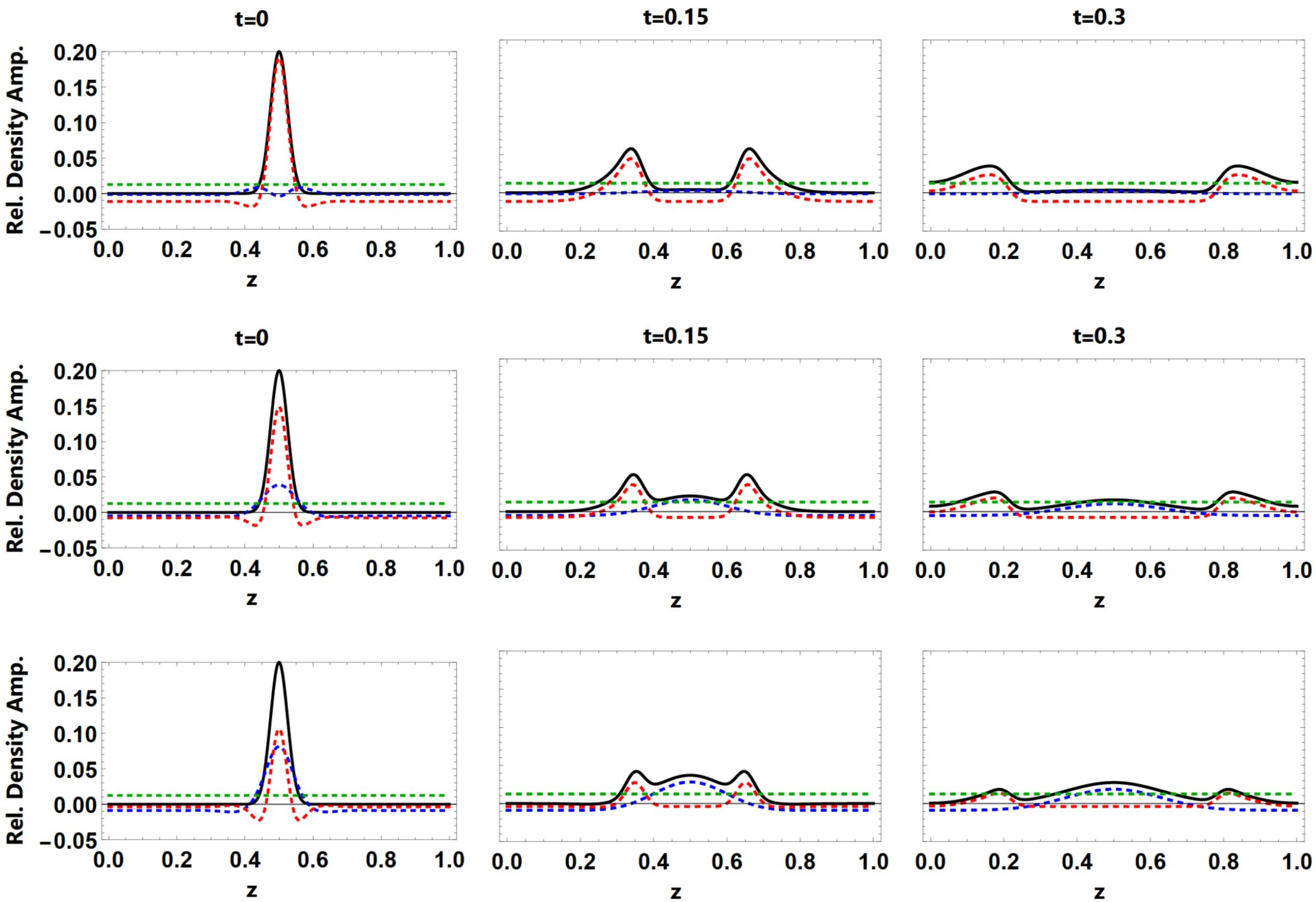
We can estimate **distribution** of perturbation full energy **between modes!!!**



Different distribution -> Different Evolution

$$a_{\rho,in}(z,0) = A_\rho \exp\left[-(z-z_0)^2/w\right], \quad a_{P,in}(z,0) = A_P \exp\left[-(z-z_0)^2/w\right],$$
$$a_{T,in}(z,0) = a_{P,in}(z,0) - a_{P,in}(z,0), \quad a_{u,in}(z,0) = 0. \tag{21}$$

Here, A_ρ and A_P are dimensionless magnitudes of the density and pressure variations; w and z_0 are the effective width and position of the perturbing pulse, respectively.



$\tau_{\text{cond}} = 25.$

A_P/A_ρ equals $1.5, 1$ and $0.5,$

Fitting observations

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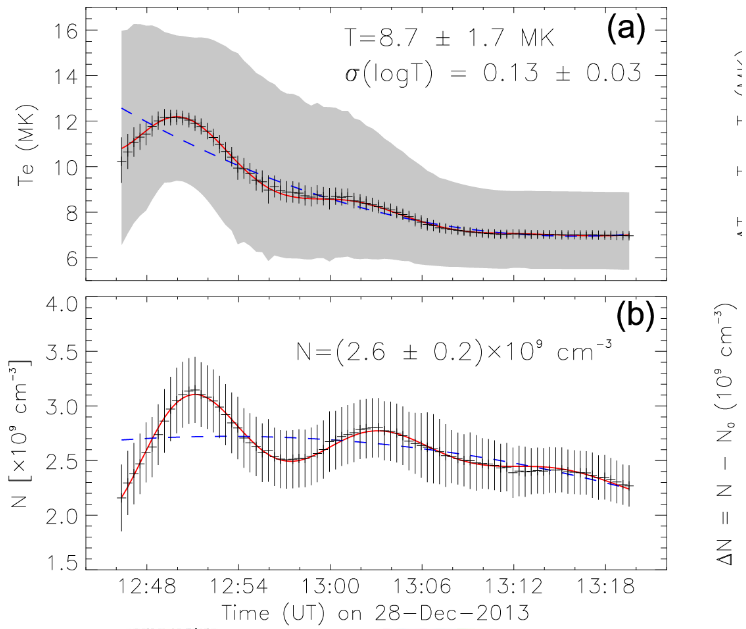
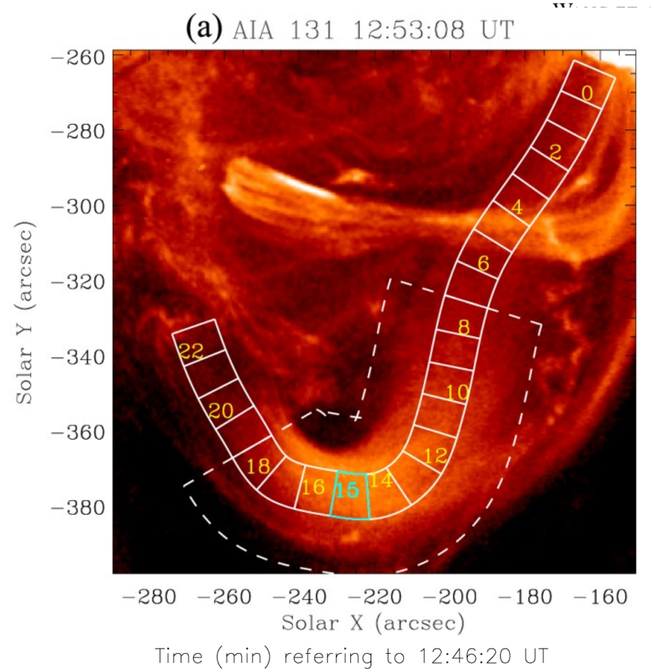
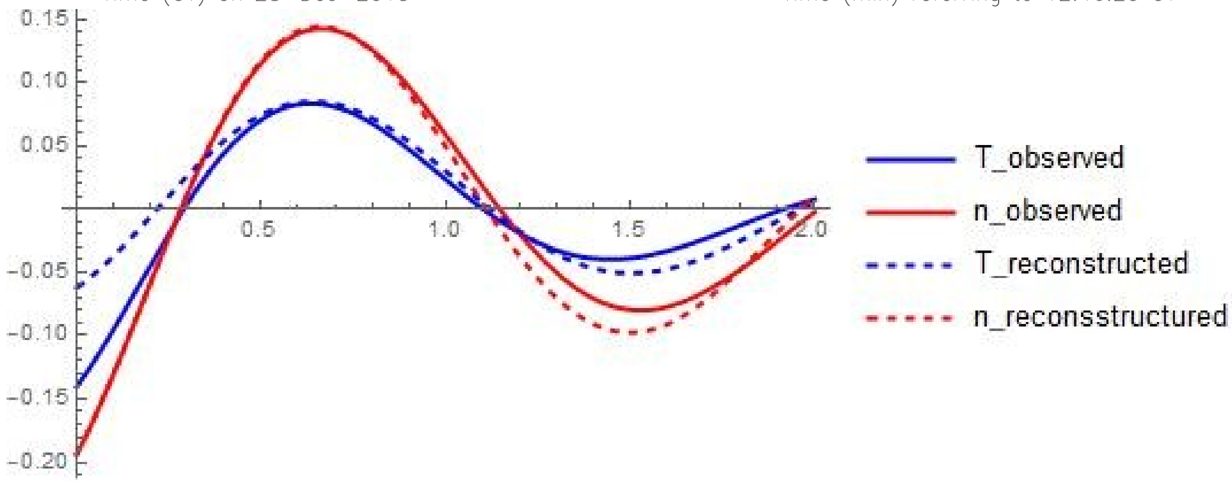






Expression for phase shifts and amplitudes

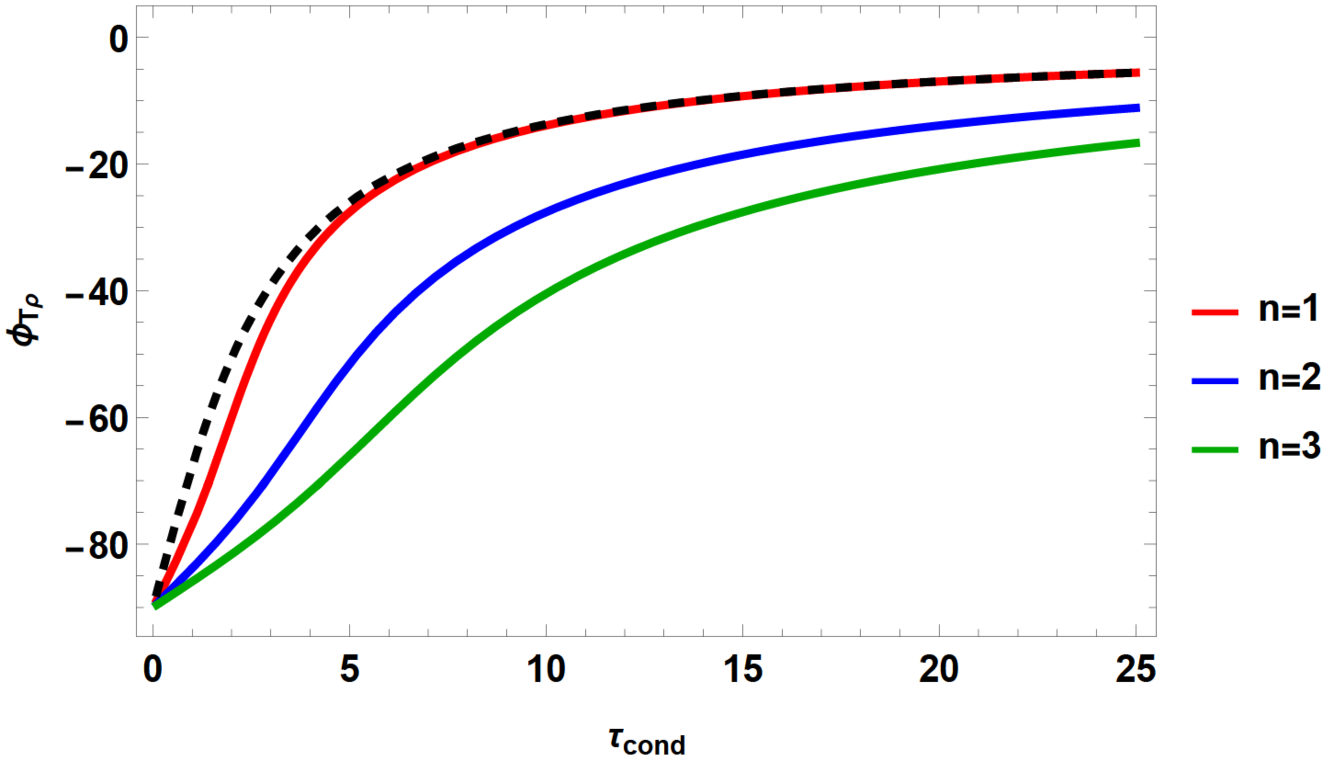
$$\phi_{T\rho} = \phi_{Tn} - \phi_{\rho n} = \arctan\left(\frac{-\left(\omega_{\mathrm{AI}}^2 + \omega_{\mathrm{AR}}^2\right)\sin 2\phi_{\rho \mathrm{u}}}{\left(\omega_{\mathrm{AI}}^2 + \omega_{\mathrm{AR}}^2\right)\cos 2\phi_{\rho \mathrm{u}} + k^2}\right).$$

$$\phi_{\rho u} = \phi_{\rho n} - \phi_{un} = \arctan\left(\frac{-\omega_{\mathrm{AR}}}{\omega_{\mathrm{AI}}}\right),$$

$$C_{0Tn} = C_{0\rho n}\sqrt{\left(\frac{\omega_{\mathrm{AI}}^2 + \omega_{\mathrm{AR}}^2}{k^2}\right)^2 + 2\left(\frac{\omega_{\mathrm{AI}}^2 + \omega_{\mathrm{AR}}^2}{k^2}\right)\cos 2\phi_{\rho \mathrm{u}} + 1}$$

$$C_{0\rho n} = C_{0un}\frac{k}{\sqrt{\omega_{\mathrm{AI}}^2 + \omega_{\mathrm{AR}}^2}}$$

Phase shifts of the first three harmonics



The way to apply theoretical model

Observed signal

- Intensity (in different channels)
- Doppler shifts
- Phase shifts between ρ, T, V

Fitting

General solution of the linear equation

- Thermal conductivity coefficient
- Mode composition of the original signal
- Constraints of the heating function
- etc

САМАРСКИЙ УНИВЕРСИТЕТ
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THANK YOU

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