The Mach cone in inhomogeneous magnetosphere: fast magnetoacoustic mode generation by the solar wind impulse oblique impact on the magnetopause

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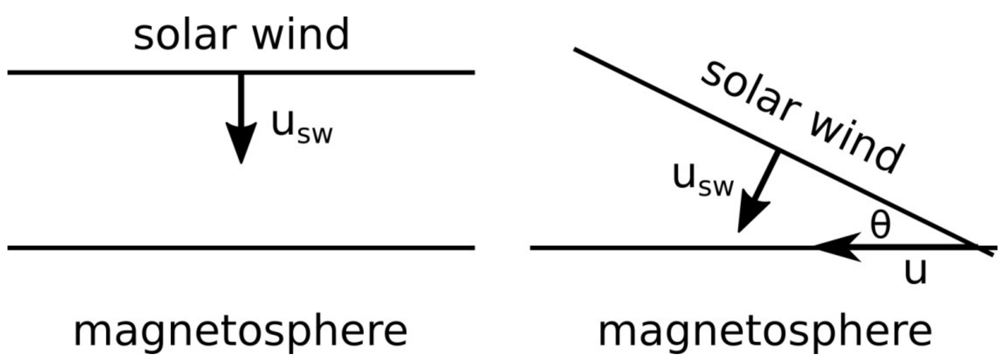
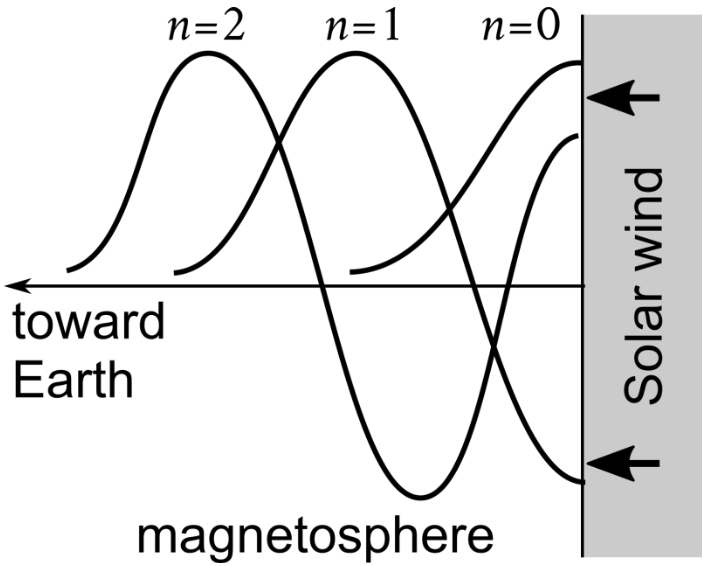
Impulsive shock on the magnetopause



- Many authors: the MHD waves in the magnetosphere are generated by the solar wind shock on the magnetopause.
- Theory (usually): the normal fall of the shock onto the magnetopause.
- More common situation: the shock is inclined.
- The impulse is running on the magnetopause.
- Two cases:
  - ~~$u \ll v_A$: impulse → the Alfven mode~~
  - $u \gg v_A$: impulse → the fast magnetoacoustic mode

Impulsive shock: normal fall

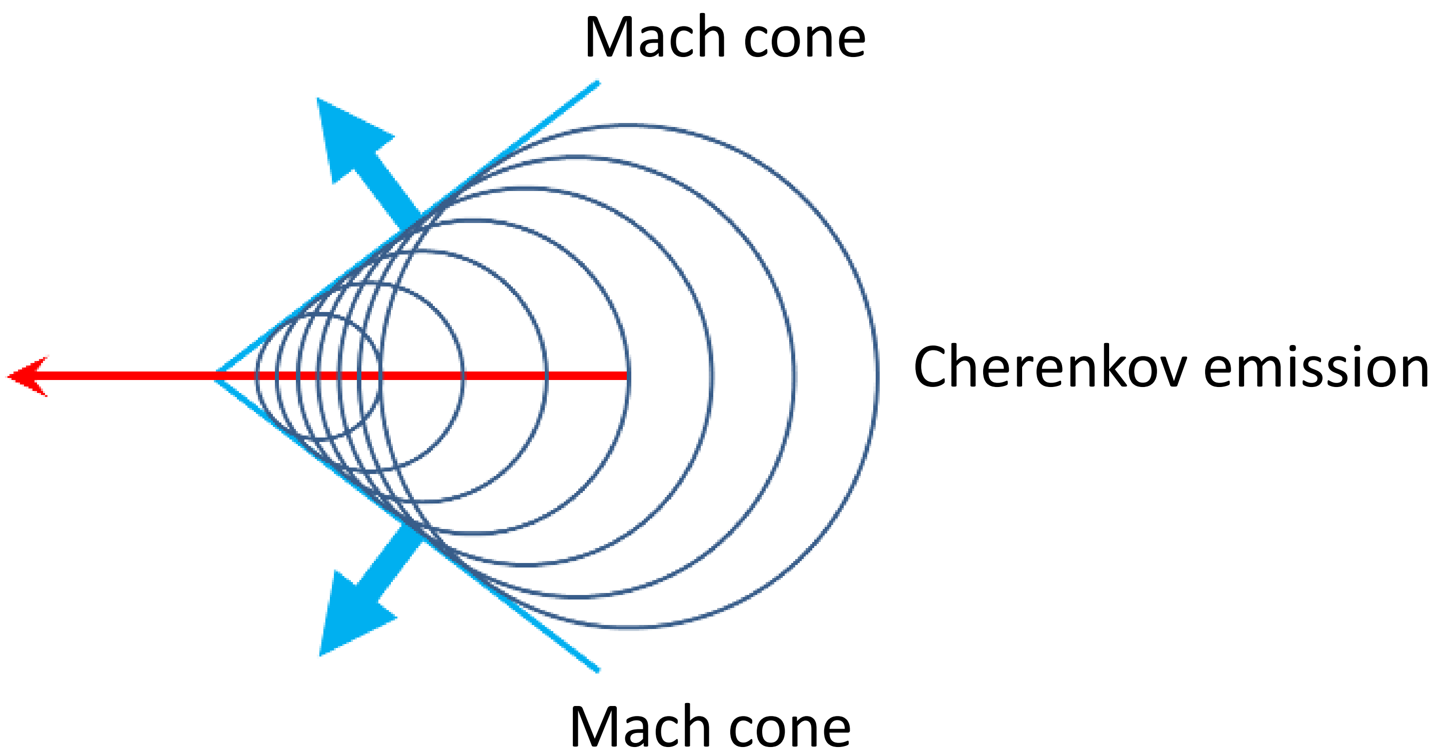


Fast mode dispersion relation:

$$k_x^2 = \frac{\omega^2}{v_A^2} - k_y^2 - k_z^2$$

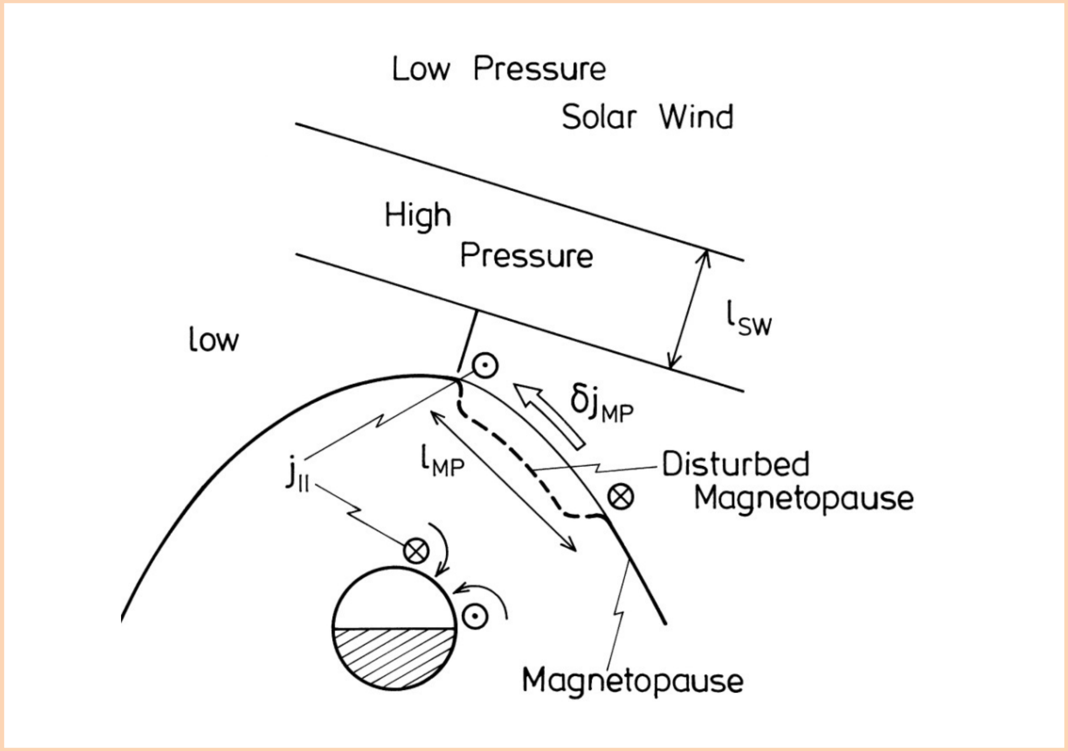
- The Alfven speed grow toward the Earth
- Discrete harmonics of cavity mode are formed

Moving source ($u \gg v_A$): homogeneous plasma



- What happens in inhomogeneous plasma?

The scenario

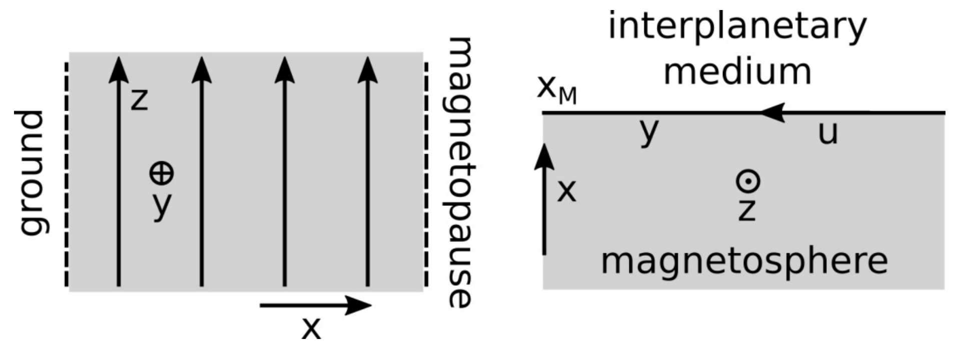


Impulse →

→ Chapman-Ferraro current perturbation →

→ the MHD wave

The model and equations



Box model

Fast mode equation:

$$B_z'' + \frac{(v_A^2)'}{v_A^2} B_z' + \left(\frac{1}{v_A^2} \frac{d^2}{dt^2} + k_y^2 + k_z^2 \right) B_z = 0$$

$$B_z = \frac{4\pi}{c} I_0 \delta(y - ut) e^{i k_z z},$$

Moving source

Equations (Fourier)

Given Fourier harmonic:

$$\tilde{B}_z'' + \frac{(v_A^2)'}{v_A^2}\tilde{B}_z' + \left(\frac{\omega^2}{v_A^2} - k_y^2 - k_z^2\right)\tilde{B}_z = 0$$

$$\tilde{B}_z = \frac{2I_0}{c}\delta(\omega - k_y u)e^{ik_z z}.$$

Solution:

$$\tilde{B}_z(x, k_y, \omega, z) = A\delta(\omega - k_y u)\frac{\sin\left(\psi_0 - \psi + \frac{\pi}{4}\right)}{\sin\left(\psi_0 + \frac{\pi}{4}\right)}$$

Designations:

$$A = \frac{2I_0}{c}\sqrt{\frac{k_{xM}}{k_x}\frac{v_{AM}}{v_A}}e^{ik_z z}, \quad \psi = \int_x^{x_M} k_x dx',$$

Solution (time dependent)

Summation over the Fourier harmonics:

$$B_z(x,y,t) = \int_{-\infty}^{\infty} dk_y e^{ik_y y} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \tilde{B}_z(x,k_y,\omega)$$

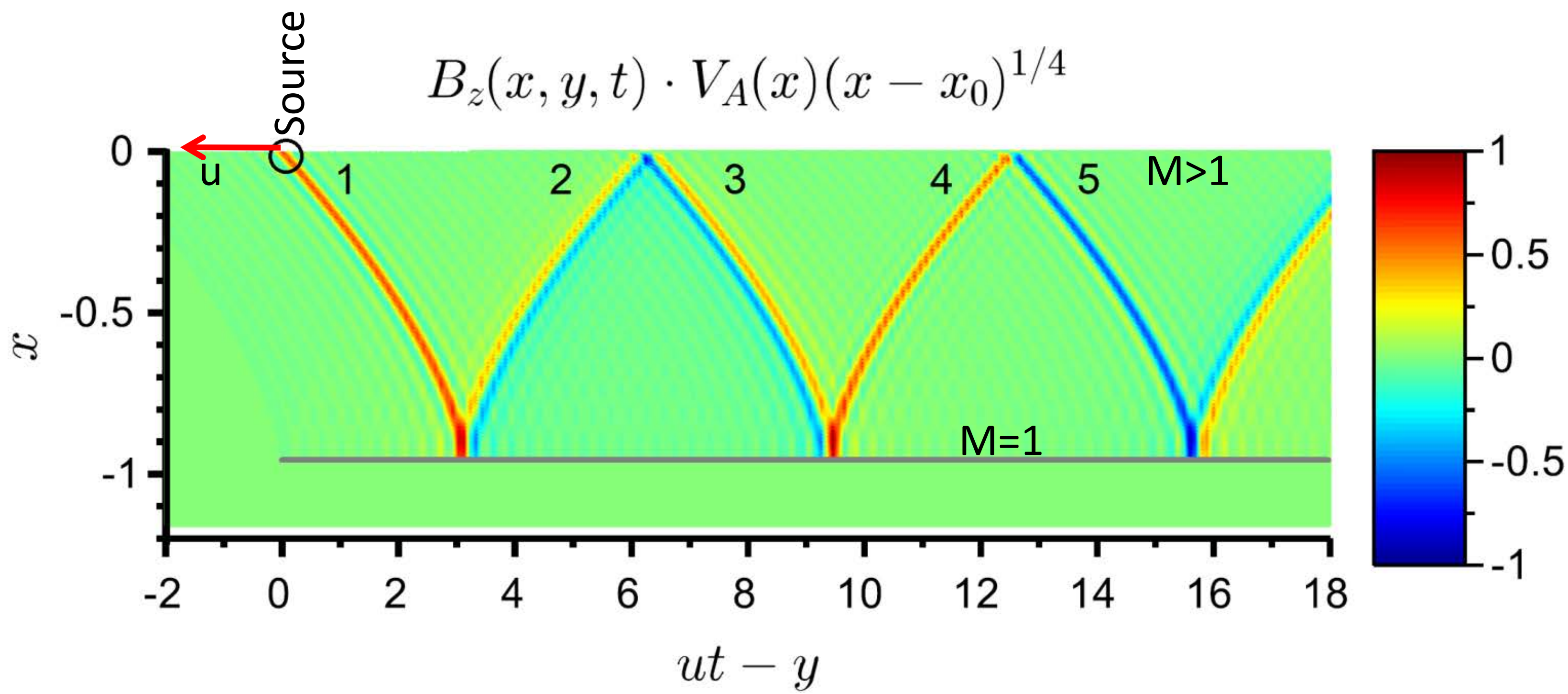
After integrations:

$$B_z = 2\pi A \cdot \sum_{n=1}^{\infty} \left\{ \Theta(\xi_0 - \xi + \tau) \cos\left(\pi n \frac{\tau - \xi}{\xi_0} - \frac{\pi}{4} \frac{\tau - \xi}{\xi_0}\right) - \Theta(\tau - \xi_0 + \xi) \cos\left(\pi n \frac{\tau + \xi}{\xi_0} - \frac{\pi}{4} \frac{\tau + \xi}{\xi_0}\right) \right\}.$$

Designations:

$$\xi = \int_{x}^{x_M} dx' \sqrt{\frac{u^2}{v_A^2} - 1} \qquad \tau = ut - y$$

Fast source ($u >> v_A$): inhomogeneous plasma



The moving source generates the Mach cone expanding with the local Alfvén speed. The Mach cone is reflected from the surface where the local Mach number equals 1. The Mach cone expands outside and reflects from the magnetopause. Reflections from the reflection surface and the magnetopause leads to turning the Mach cone into the curved polyline.

Thank you!